# ANNUAL EXAMINATION 2020

**(Only for Regular Students)**

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| ***Centre No. 135*** | ***Centre Name- Disha College, Raipur (C.G.)*** |
| **Class-B.Sc.-II** | **Subject- Mathematics** |
| **Paper No- I** | **Paper Name-Advanced Calculus** |
| **Time- 3 hrs.** | **M.M.-50** |

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**Note:- All questions are compulsory. Solve any two parts of each question. All question carry equal marks.**

**UNIT-I**

Q1(a) **n'kkb; dh vuØe** $\{a_n\}_{n=1}^{\infty}$ **tgk¡** $a_n = 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}$

**vfHklkjh gSA**

Prove that the sequence $\{a_n\}_{n=1}^{\infty}$ where $a_n = 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}$

is convergent sequence.

(b) **fuEufyf[kr Js.kh dh vfHklkfjrk ;k vilkfjrk dk ijh{k.k dhft,%**

Test the convergence of the following series:

$$1 + \frac{x}{2} + \frac{2!}{3^2}x^2 + \frac{3!}{4^3}x^3 + \frac{4!}{5^4}x^4 + \cdots, x > 0$$

(c) **fl) dhft, fd çR;sd fujis{k vfHkljh Js.kh ,d vfHkljh Js.kh gksrk g fdUrq blds foykse lR; ugh gSA**

Prove that every absolutely convergence series is convergent but not conversely.

**UNIT-II**

Q2(a) **jksy çes; dk fn, x, Qyu d fy, lR;kfir dhft,%**

$$f(x) = x^3 - 6x^2 + 11x - 6$$

Verify Rolle's theorem for the function:

$$f(x) = x^3 - 6x^2 + 11x - 6$$

(b) $f(x) = \frac{x}{1+e^{1/x}}$ **tc** $x \neq 0$ **,o** $f(0) = 0$, **rks fl) dhft, fd** $f(x)$ **ij lrr g ijUrq vodyuh; ughA**

If $f(x) = \frac{x}{1+e^{1/x}}$ when $x \neq 0$ and $f(0) = 0$, then show that $f(x)$ is continuous

but not differentiable at x = 0

(c) **Qyu** f(x) = logx **d fy, vUrjky** [1,e] **e ysxkt e/;eku çes; dk lR;kfir dhft,A**

Verify Lagrange's Mean value theorem for the function f(x) = logx in the interval [1,e].

## UNIT-III

Q3(a) **fl) dhft, fd Qyu** $f(x,y) = \frac{xy}{x^2+y^2}$ $(x,y) \neq 0{,}0$ $\lim_{(x,y)\to(o,o)} f(x,y)$ **fo|eku ugh g] ijUrq iqujkdr lhek,¡ cjkcj gSA**

Show that : $f(x,y) = \frac{xy}{x^2+y^2}$ for $(x,y) \neq 0{,}0$ $\lim_{(x,y)\to(o,o)} f(x,y)$ doesn't exist where as iterated limits are equal.

(b) **;fn** $u = e^{xyz}$, **rks n'kkZb,, fd%&**

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$$

If $u = e^{xyz}$, then show that:

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$$

(c) **Qyu** : $f(x,y) = x^2 + xy + y^2$ **dk** (x-2) and (y-3) **vkj d ?kkrk e Vyj çlkj Kkr dhft,A**

Expand the function: $f(x,y) = x^2 + xy + y^2$ by Taylor's expansion in power of (x-2) and (y-3)

## UNIT-IV

Q4(a) **ljy js[kkvks** x cos ∝ + y sin ∝ = l sin∝ . cos∝ **ds dqy dk vUokyksi Kkr dhft, tgk¡ dks.k ∝ çkpy gSA**

Find the envelope of the family of straight lines : x cos ∝ + y sin ∝ = l sin∝ . cos∝ where angle ∝ is a parameter.

(b) **Qyu ds mfPp"B vFkok fufEu"B eku dh foospuk dhft,A**

$$u = xy + \frac{a^3}{x} + \frac{a^3}{y}$$

Discuss the minimum or maximum value of the function:

$$u = xy + \frac{a^3}{x} + \frac{a^3}{y}$$

(c) **vfoijoy;** $2xy = a^2$ **dk dUnt Kkr dhft,A**

Find the evolutes of the hyperbola $2xy = a^2$

## UNIT-V

Q5(a) **fl) dhft,**: $\beta(m,n) = \frac{\Gamma m \,.\, \Gamma n}{\Gamma(m+n)}$ $(m,n) > 0$

Prove that: $\beta(m,n) = \frac{\Gamma m \,.\, \Gamma n}{\Gamma(m+n)}$ $(m,n) > 0$

(b) **flekdy dk Øe ifjorlu dhft,%**

$$\int_0^{2a} \int_{x^2/4a}^{3a-x} f(x, y)\,dx\,dy$$

Change the order of integration in the double integral:

$$\int_0^{2a} \int_{x^2/4a}^{3a-x} f(x, y)\,dx\,dy$$

(c) **eku fudkfy,**:

$$\int_0^{2} \int_0^{x} \int_0^{x+y} e^x(y + 2z)\,dx\,dy\,dz$$

Evaluate:

$$\int_0^{2} \int_0^{x} \int_0^{x+y} e^x(y + 2z)\,dx\,dy\,dz$$

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